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CRITICAL SHEAR VISCOSITY EXPONENT: EXPERIMENT AND THEORY

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ABSTRACT

Static and dynamic responses can become singular near the critical point, because of the long ranged fluctuations in the system. In a gas-liquid system near the critical point or a binary liquid near the consolute point, this leads to divergent transport properties at the critical temperature. While the thermal conductivity and bulk viscosity exhibit strong divergence, the shear viscosity exhibits a weak divergence. The divergence of shear viscosity, being characterised by a small exponent x_η, requires careful theoretical and experimental study. Since experiments, carried out in space shuttle, are not plagued by gravity induced stratification, it is possible to approach the critical point closely and thus carry out an accurate determination of the small exponent. The precise measurement demands a careful calculation. In this article, we will try to elaborate on the experimental and theoretical progress done so far on the critical shear viscosity exponent x_η.

INTRODUCTION

Critical exponents, amplitude ratios and scaling functions were issues of considerable importance three decades ago. Sophisticated calculations and experiments were carried out which clearly established the correctness of various theoretical models (Landau-Ginzburg equations for statics and various models of dynamics[1-4] introduced by Hohenberg and Halperin, Kawasaki, Ferrell etc). Basically, the exponents could be classified into two types: i) large exponents i.e. exponents of O(1) and ii) small exponents i.e. exponents of O(0.1) or even smaller. It is the small exponents where the most crucial confrontation between theory and experiments can occur. That is why even after three decades the small exponents remain an interesting issue. In static critical phenomena [5] the small exponents are associated with the critical correlation function at the transition point (η, the anomalous dimension exponent) and specific heat (α, the specific heat exponent, the specific heat at constant volume for the liquid-gas transition and the specific heat at constant pressure for the superfluid transition of He^4), while in the critical dynamics the small exponent is associated with the shear viscosity. This weak divergence of the shear viscosity is characterized by a small exponent, x_η through the relation:

$$\eta \propto \xi^{x_\eta} \propto \kappa^{-x_\eta} \tag{1}$$

where $\xi = \kappa^{-1}$ is the correlation length. $\kappa \to 0$ as $T \to T_c$. The recent measurements [6] in the space shuttle have yielded an accurate value, namely

$$x_\eta = 0.0690 \pm 0.0006 \tag{2}$$

Let us now proceed to the study this small shear viscosity exponent x_η theoretically. In a liquid-gas system near the critical point or a binary liquid mixture near the critical mixing point, the order parameter ψ is the density (concentration) difference and relaxes when disturbed from equilibrium according to the Langevin equation

$$\frac{\partial\psi(\vec{k})}{\partial t} = -\Gamma k^2(k^2+\kappa^2)\psi(\vec{k}) + N(\vec{k}) \tag{3}$$

Where $\psi(\vec{k})$ is the Fourier transformation of the D-dimensional field $\psi(\vec{x_1}, \dots .\vec{x_D})$. In the relaxation rate the factor k^2 indicates that the field ψ is conserved. Γ is the Onsagar coefficient and the diffusion constant is $D = \frac{\Gamma}{\chi}$, where χ is the susceptibility. Near the critical point, the susceptibility is $\chi = \frac{1}{(k^2+\kappa^2)}$ with $\kappa = \xi^{-1}$, the inverse correlation length, which diverges near $T = T_c$ as$\xi \propto |T - T_c|^{-\nu}$. The term N is a stochastic force that comes from the short wavelength modes. Fluctuation dissipation holds and the correlation of N is related the usual way to the dissipation.

In a fluid the density (concentrations) fluctuations will be affected by the velocity fluctuations and the effect of the velocity is to advect the concentration field, so that

$$\frac{\partial\psi(\vec{k})}{\partial t} + ik_\alpha \sum_{\vec{p}} v_\alpha(\vec{p})\psi(\vec{k}-\vec{p}) = -\Gamma k^2(k^2+\kappa^2)\psi(\vec{k}) + N(\vec{k}) \tag{4}$$

The fact that the velocity fluctuations affect the concentration means that we need to know the velocity fluctuations. The equation of motion (for small fluctuations) is Navier-Stokes equation

$$\frac{\partial v_\alpha(\vec{k})}{\partial t} = -\eta k^2 v_\alpha(\vec{k}) + N_\alpha^v(\vec{k}) \tag{5}$$

We note that v_α and N_α are solenoidal. However Eqs.(4) and (5) do not conserve the local free energy density $\sum_{\vec{k}}[(k^2+\kappa^2)\psi(\vec{k})\psi(-\vec{k}) + v(\vec{k})v(-\vec{k})]$ when the dissipation terms are omitted and consequently Eq.(5) needs to be argumented as

$$\frac{\partial v_\alpha(\vec{k})}{\partial t} + i\sum_{\vec{p}} p^2 p_\beta \mathrm{T}_{\alpha\beta}(\vec{k})\psi(\vec{p})\,\psi(\vec{k}-\vec{p}) = -\eta k^2 v_\alpha(\vec{k}) + N_\alpha^v(\vec{k}) \tag{6}$$

where $\mathrm{T}_{\alpha\beta}(k) = \delta_{\alpha\beta} - \frac{k_\alpha\beta}{k^2}$, the projection operator.

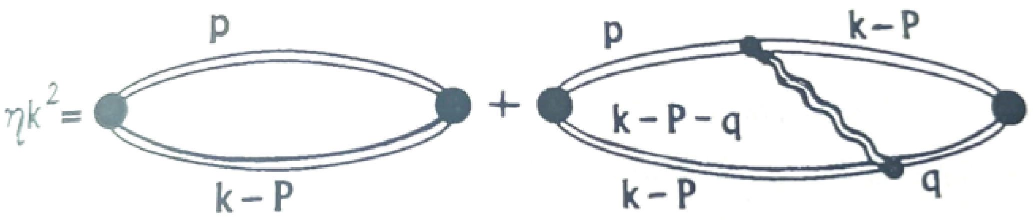

Figure 1: Diagrammatic expansion of shear viscosity to two loop order

Now, in the one-loop self consistent calculation, the frequency dependent shear viscosity in $D = 3$ is given by (Fig.1)

$$\Delta\eta_1(\kappa,\omega) = \frac{1}{4}\int \frac{d^3p}{(2\pi)^3}\frac{4p^4 sin^2\theta cos^2\theta}{(p^2+\kappa^2)^2\left[-i\omega+2\Gamma_0 p^2(p^2+\kappa^2)^{1/2}\right]}$$

$$= \frac{1}{30\pi^2\Gamma_0}\int \frac{p^6 dp}{(p^2+\kappa^2)^2\left[\frac{-i\omega}{2\Gamma_0}+p^2(p^2+\kappa^2)^{1/2}\right]} \tag{7}$$

In the hydrodynamic limit (zero frequency), we get from Eq.(7)

$$\Delta\eta_1(\kappa,\omega=0)=\frac{1}{30\pi^2\Gamma_0}\int\frac{p^4dp}{(p^2+\kappa^2)^{5/2}}$$

$$=\frac{1}{30\pi^2\Gamma_0}\left[ln\frac{\Lambda}{\kappa}+ln2-\frac{4}{3}\right]$$ (Λ is the ultra-violet cut-off)

$$=\frac{\eta_0}{30\pi^2(\eta_0\Gamma_0)}\left[ln\frac{\Lambda}{\kappa}+ln2-\frac{4}{3}\right]$$ (η_0 is the non-critical background value of the viscosity)

The product $\eta_0\Gamma_0$ is fixed by the diffusion coefficient diagrammatics and is $\frac{1}{16}$. Therefore

$$\Delta\eta_1(\kappa,\omega=0)=\frac{8}{15\pi^2}\eta_0\left[ln\frac{\Lambda}{\kappa}+ln2-\frac{4}{3}\right] \tag{8}$$

So, up to one loop order

$$\eta(\kappa)=\eta_0+\Delta\eta_1(\kappa)=\eta_0+\frac{8}{15\pi^2}\eta_0\left[ln\frac{\Lambda}{\kappa}+ln2-\frac{4}{3}\right] \tag{9}$$

Comparing Eqs. (1) and (9)

$$x_\eta=\frac{8}{15\pi^2}\cong 0.054 \tag{10}$$

From Eqs. (2) and (10), we see that the one loop answer is about 20% away from the experimental value. So, we go for the theoretical self consistent two loop calculation in $D=3$.

The two loop contribution to the shear viscosity η (Fig.1) is given by the vertex correction diagram and can be written as

$$\Delta\eta_2k^2=\frac{1}{D-1}\int\frac{d^Dp}{(2\pi)^D}\int\frac{d^Dq}{(2\pi)^D}\frac{\left[p^2-(\vec{k}-\vec{p})^2\right]\left[q^2-(\vec{k}-\vec{q})^2\right][p_\alpha T_{\alpha\beta}(\vec{k})q_\beta][p_\alpha T_{\alpha\beta}(\vec{k}-\vec{p}-\vec{q})q_\beta]}{\eta_0(p^2+\kappa^2)(q^2+\kappa^2)|\vec{p}+\vec{q}-\vec{k}|^2[-i\omega+\Gamma(\vec{p},\kappa)+\Gamma(\vec{k}-\vec{p},\kappa)][-i\omega+\Gamma(\vec{q},\kappa)+\Gamma(\vec{k}-\vec{q},\kappa)]} \tag{11}$$

In the above $\Gamma(\vec{k},\kappa)$ is the fully dressed order parameter relaxation rate. From the right hand side, we need to extract the $O(k^2)$ term. We also need to average over all possible directions of $\vec{k}$.

Accordingly,

$$p^2-(\vec{k}-\vec{p})^2\cong-2\vec{k}\cdot\vec{p}$$

$$q^2-(\vec{k}-\vec{q})^2\cong-2\vec{k}\cdot\vec{q}$$

and

$$\langle(\vec{k}\cdot\vec{p})(\vec{k}\cdot\vec{q})p_\alpha T_{\alpha\beta}(\vec{k})q_\beta\rangle=\frac{k^2p^2q^2(Dcos^2\theta-1)}{D(D+2)}$$

Everywhere else in the right hand side, we may set $k=0$ in Eq.(11). Thus the directional average of $p_\alpha T_{\alpha\beta}(\vec{k}-\vec{p}-\vec{q})q_\beta$ becomes $\langle p_\alpha T_{\alpha\beta}(\vec{p}+\vec{q})q_\beta\rangle$. Accordingly,

$$\langle p_\alpha \mathrm{T}_{\alpha\beta}(\vec{p}+\vec{q})q_\beta\rangle = -\frac{p^2q^2sin^2\theta}{(\vec{p}+\vec{q})^2} \tag{12}$$

We can now write Eq.(11) as

$$\Delta\eta_2 = \frac{1}{(D-1)D(D+1)\eta_0}\int\frac{d^Dp}{(2\pi)^D}\int\frac{d^Dq}{(2\pi)^D}\frac{p^4q^4sin^2\theta[1-Dcos^2\theta]}{(p^2+\kappa^2)(q^2+\kappa^2)(\vec{p}+\vec{q})^4}\times\frac{1}{[-i\omega+2\Gamma(\vec{p},\kappa)][-i\omega+2\Gamma(\vec{q},\kappa)]} \tag{13}$$

Specializing to $D = 3$, we can replace the relaxation rate $\Gamma(\vec{k},\kappa)$ by an accurate approximate to the full Kawasaki function as $\Gamma(\vec{k},\kappa) = \Gamma_0 k^2(k^2+\kappa^2)^{1/2}$ and then we have

$$\Delta\eta_2(\kappa,\omega) = \frac{\eta_0}{30(\eta_0\Gamma_0)(\eta_0\Gamma_0)}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\frac{p^4q^4sin^2\theta[1-3cos^2\theta]}{(p^2+\kappa^2)(q^2+\kappa^2)(\vec{p}+\vec{q})^4}\times\frac{1}{\left[-\frac{i\omega}{2\Gamma_0}+p^2(p^2+\kappa^2)^{1/2}\right]\left[-\frac{i\omega}{2\Gamma_0}+q^2(q^2+\kappa^2)^{1/2}\right]}$$

In the hydrodynamic limit $\omega = 0$ and then

$$\Delta\eta_2(\kappa,\omega=0) = \frac{\eta_0}{30(\eta_0\Gamma_0)(\eta_0\Gamma_0)}\frac{1}{(2\pi)^6}\int\frac{p^2q^2sin^2\theta[1-3cos^2\theta]d^3pd^3q}{(p^2+\kappa^2)^{3/2}(q^2+\kappa^2)^{3/2}(\vec{p}+\vec{q})^4} = \frac{\eta_0}{30(\eta_0\Gamma_0)(\eta_0\Gamma_0)}\frac{1}{(2\pi)^6}I \tag{14}$$

where

$$I = \int\frac{p^2q^2sin^2\theta[1-3cos^2\theta]d^3pd^3q}{(p^2+\kappa^2)^{3/2}(q^2+\kappa^2)^{3/2}(\vec{p}+\vec{q})^4} = \int\frac{p^2d^3p}{(p^2+\kappa^2)^{3/2}}I_1 \tag{15}$$

where

$$I_1 = \int\frac{q^2sin^2\theta[1-3cos^2\theta]d^3q}{(q^2+\kappa^2)^{3/2}(\vec{p}+\vec{q})^4} \tag{16}$$

For $p \to 0$

$$I_1 \to \int\frac{q^2sin^2\theta[1-3cos^2\theta]d^3q}{(q^2+\kappa^2)^{3/2}q^4} = 2\pi\int_0^\infty\frac{dq}{(q^2+\kappa^2)^{3/2}}\int_0^\pi(1-3cos^2\theta)sin^3\theta d\theta = 2\pi\times\frac{1}{\kappa^2}\times\frac{8}{15} = \frac{16\pi}{15\kappa^2} \tag{17}$$

For $p \to \infty$

$$I_1 \to \int\frac{q^2sin^2\theta[1-3cos^2\theta]d^3q}{q^3(\vec{p}+\vec{q})^4} = \frac{2\pi}{p^2}\int_0^\pi d\theta\,(1-3cos^2\theta)sin^3\theta\int_0^\infty dx\frac{x}{(x^2+2xcos\theta+1)^2} =$$
$$\int_0^\pi d\theta\,(1-3cos^2\theta)sin^3\theta\times\frac{1}{2sin^2\theta}\left[1-\left(\frac{\pi}{2}-\theta\right)cot\theta\right] = \frac{8\pi}{3p^2} \tag{18}$$

By the method of interpolation, we approximately write I_1 as

$$I_1 \cong \frac{8\pi}{3}\left(p^2+\frac{5}{2}\kappa^2\right)^{-1} \tag{19}$$

Using Eq.(19) in (15), we have after a little algebra

$$I = \frac{32\pi^2}{3}\int_0^\infty\frac{p^4dp}{(p^2+\kappa^2)^{3/2}\left(p^2+\frac{5}{2}\kappa^2\right)} = \frac{32\pi^2}{3}\left[ln\frac{\Lambda}{\kappa}-0.865\right] \tag{20}$$

Finally, using Eq.(20) in (14) and $\eta_0\Gamma_0 = \frac{1}{16}$, we get

$$\Delta\eta_2(\kappa) = \eta_0\frac{8}{15\pi^2}\frac{8}{3\pi^2}\left[ln\frac{\Lambda}{\kappa}-0.865\right] \tag{21}$$

Therefore, up to two loop order

$$\eta(\kappa) = \eta_0 + \Delta\eta_1 + \Delta\eta_2 = \eta_0 + \eta_0 \frac{8}{15\pi^2}\left(1 + \frac{8}{3\pi^2}\right) ln\frac{\Lambda}{\kappa} + \cdots \quad (22)$$

Comparing with Eq.(1)

$$x_\eta = \frac{8}{15\pi^2}\left(1 + \frac{8}{3\pi^2}\right) \cong 0.0685 \quad (23)$$

This value of x_η is within 2% (the correction coming from two loop self energy insertion graphs and the dissipative four point coupling) of the experimental value [Eq.(2)].This raises the immediate question: What happened to the higher loops that have been left out? In the subsequent discussion, we shall show through detailed calculation that the higher loops contributions are vanishingly small.

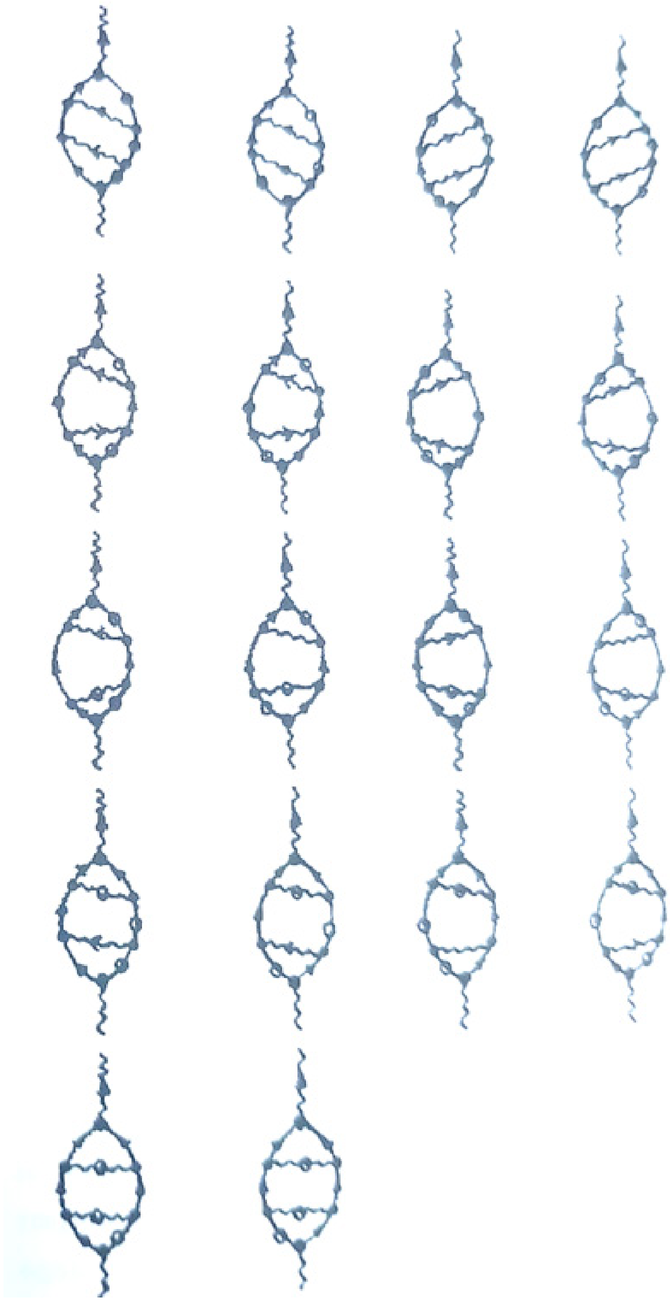

Figure 2: Three loop graphs.

The three loop diagrams of the vertex correction variety for the viscosity showing all possible time orderings are shown in Fig. (2). Propagators appear with an arrow and correlators with an open circle. We consider a typical diagram (diagram no.5 of Fig.2). The corresponding self-energy is

$$\Sigma_j(\vec{k},\kappa,\omega) = -\frac{1}{8(D-1)}\int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{d^Dr}{(2\pi)^D}\, d\nu_1 d\nu_2 d\nu_3 \left[\left(\vec{k}-\vec{p}\right)^2 - p^2\right]\left[\left(\vec{k}-\vec{r}\right)^2 - r^2\right]\left[p_\alpha \mathrm{T}_{\alpha\beta}(\vec{k})r_\beta\right]\left[p_\mu \mathrm{T}_{\mu\nu}(\vec{k}-\vec{p}-\vec{q})q_\nu\right]\left[q_\gamma \mathrm{T}_{\gamma\lambda}(\vec{k}-\vec{q}-\vec{r})r_\lambda\right]G_{0\psi}(\vec{k}-\vec{p},\omega-\nu_1)C_{0\psi}(\vec{p},\nu_1)G_{0j}(\vec{k}-\vec{p}-\vec{q},\omega-\nu_1-\nu_2)G_{0\psi}(\vec{k}-\vec{q},\omega-\nu_2)C_{0\psi}(\vec{q},\nu_2)G_{0j}(\vec{k}-\vec{q}-\vec{r},\omega-\nu_2-\nu_3)G_{0\psi}(\vec{k}-\vec{r},\omega-\nu_3)C_{0\psi}(\vec{r},\nu_3)$$

$$= -\frac{1}{8(D-1)}\int \frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{d^Dr}{(2\pi)^D}$$

$$\frac{\left[(\vec{k}-\vec{p})^2-p^2\right]\left[(\vec{k}-\vec{r})^2-r^2\right]\left[p_\alpha \mathrm{T}_{\alpha\beta}(\vec{k})r_\beta\right]\left[p_\mu \mathrm{T}_{\mu\nu}(\vec{k}-\vec{p}-\vec{q})q_\nu\right]\left[q_\gamma \mathrm{T}_{\gamma\lambda}(\vec{k}-\vec{q}-\vec{r})r_\lambda\right]}{\left[(\vec{k}-\vec{p})^2+\kappa^2\right]\left[(\vec{k}-\vec{q})^2+\kappa^2\right]\left[(\vec{k}-\vec{r})^2+\kappa^2\right]}\times$$

$$\frac{1}{\left[-i\omega+\lambda p^2(p^2+\kappa^2)+\lambda(\vec{k}-\vec{p})^2\left((\vec{k}-\vec{p})^2+\kappa^2\right)\right]}\times$$

$$\frac{1}{\left[-i\omega+\lambda q^2(q^2+\kappa^2)+\lambda(\vec{k}-\vec{q})^2\left((\vec{k}-\vec{q})^2+\kappa^2\right)\right]}\times$$

$$\frac{1}{\left[-i\omega+\lambda r^2(r^2+\kappa^2)+\lambda(\vec{k}-\vec{r})^2\left((\vec{k}-\vec{r})^2+\kappa^2\right)\right]\eta_0(\vec{k}-\vec{p}-\vec{q})^2\eta_0(\vec{k}-\vec{q}-\vec{r})^2} \qquad (24)$$

We now use $\lambda(k,\kappa) = \frac{\Gamma_0}{(k^2+\kappa^2)^{1/2}}$ and $\eta_0\Gamma_0 = \frac{1}{16}$ and also average over all directions of $\vec{k}$. After extracting a factor of k^2 from the integral, we set $k = 0$ everywhere else. We also set $D = 3$ and $\omega = 0$. Therefore,

$$\frac{\Delta\eta_3(\kappa,\omega=0)}{\eta_0} = -\frac{128}{k^2}\int \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{d^3r}{(2\pi)^3}\frac{\langle(\vec{k}\cdot\vec{p})(p_\alpha \mathrm{T}_{\alpha\beta}(\vec{k})r_\beta)(\vec{k}\cdot\vec{r})\rangle\langle[p_\mu \mathrm{T}_{\mu\nu}(\vec{p}+\vec{q})q_\nu]\rangle\langle[q_\gamma \mathrm{T}_{\gamma\lambda}(\vec{q}+\vec{r})r_\lambda]\rangle}{p^2q^2r^2(p^2+\kappa^2)^{3/2}(q^2+\kappa^2)^{3/2}(r^2+\kappa^2)^{3/2}(\vec{p}+\vec{q})^2(\vec{q}+\vec{r})^2} \qquad (25)$$

Clearly,

$$\langle(\vec{k}\cdot\vec{p})(p_\alpha \mathrm{T}_{\alpha\beta}(\vec{k})r_\beta)(\vec{k}\cdot\vec{r})\rangle = \frac{k^2p^2r^2}{15}\left[3\cos^2(\vec{p},\vec{r})-1\right]$$

$$\langle[p_\mu \mathrm{T}_{\mu\nu}(\vec{p}+\vec{q})q_\nu]\rangle = -\frac{p^2q^2\sin^2(\vec{p},\vec{q})}{(\vec{p}+\vec{q})^2}$$

$$\langle[q_\gamma \mathrm{T}_{\gamma\lambda}(\vec{q}+\vec{r})r_\lambda]\rangle = -\frac{q^2r^2\sin^2(\vec{q},\vec{r})}{(\vec{q}+\vec{r})^2}$$

Substituting these results in Eq.(25) , we have

$$\frac{\Delta\eta_3(\kappa)}{\eta_0} = -\frac{128}{15}\int \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{d^3r}{(2\pi)^3}\frac{p^2q^2r^2\sin^2(\vec{p},\vec{q})\sin^2(\vec{q},\vec{r})\left[3\cos^2(\vec{p},\vec{r})-1\right]}{(p^2+\kappa^2)^{3/2}(q^2+\kappa^2)^{3/2}(r^2+\kappa^2)^{3/2}(\vec{p}+\vec{q})^4(\vec{q}+\vec{r})^4} \qquad (26)$$

The internal momentum vectors $\vec{p}$, $\vec{q}$ and $\vec{r}$ are coupled to each other. To get rid of this complexity, we make an approximation that we find the first non-trivial contribution and then average over all directions of $\vec{q}$. Accordingly,

$$\frac{1}{(\vec{p}+\vec{q})^4} = \frac{1}{(p^2+q^2)^2}\left[1 - \frac{4(\vec{p}\cdot\vec{q})}{p^2+q^2} + \frac{6(\vec{p}\cdot\vec{q})^2}{(p^2+q^2)^2} - \cdots\right]$$

$$\frac{1}{(\vec{q}+\vec{r})^4} = \frac{1}{(q^2+r^2)^2}\left[1 - \frac{4(\vec{q}\cdot\vec{r})}{q^2+r^2} + \frac{6(\vec{q}\cdot\vec{r})^2}{(q^2+r^2)^2} - \cdots\right]$$

Therefore, the first non-trivial contribution comes from the term $\frac{36(\vec{p}\cdot\vec{q})^2(\vec{q}\cdot\vec{r})^2}{(p^2+q^2)^4(q^2+r^2)^4}$ of the expansion of $\frac{1}{(\vec{p}+\vec{q})^4(\vec{q}+\vec{r})^4}$. Also $\langle(\vec{p}\cdot\vec{q})^2(\vec{q}\cdot\vec{r})^2\rangle_{all\ directions\ of\ \vec{q}} = \frac{p^2r^2q^4}{15}[2cos^2(\vec{p},\vec{r}) + 1]$ and

$\langle[3cos^4(\vec{p},\vec{r}) - cos^2(\vec{p},\vec{r})]\rangle_{all\ directions\ of\ \vec{r}} = 3 \times \frac{3}{15} - \frac{1}{3} = \frac{4}{15}$.

Considering the above results, the first non-trivial contribution comes out to be

$$\frac{\Delta\eta_3(\kappa)}{\eta_0} = -\frac{128}{15} \times \frac{72}{15} \times \frac{4}{15} \int \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{d^3r}{(2\pi)^3}\frac{p^4q^6r^4sin^2(\vec{p},\vec{q})sin^2(\vec{q},\vec{r})}{(p^2+\kappa^2)^{3/2}(q^2+\kappa^2)^{3/2}(r^2+\kappa^2)^{3/2}(p^2+q^2)^4(q^2+r^2)^4} \tag{27}$$

Performing the angular integrals, we finally get

$$\frac{\Delta\eta_3(\kappa)}{\eta_0} = -\frac{128}{15} \times \frac{72}{15} \times \frac{4}{15} \times \left(\frac{8\pi}{3}\right)^2 \times \frac{4\pi}{(2\pi)^9} \times \int_0^\infty \frac{dpdqdrp^6q^8r^6}{(p^2+\kappa^2)^{3/2}(q^2+\kappa^2)^{3/2}(r^2+\kappa^2)^{3/2}(p^2+q^2)^4(q^2+r^2)^4}$$

$$= -\frac{6.296\times10^{-4}}{\kappa^2} \times I \tag{28}$$

Where

$$I = \int_0^\infty \frac{dpdqdrp^6q^8r^6}{(p^2+1)^{3/2}(q^2+1)^{3/2}(r^2+1)^{3/2}(p^2+q^2)^4(q^2+r^2)^4} \tag{29}$$

A Monte-Carlo simulation yields $I \sim O(10^{-3})$.This is the point that we wanted to make. Thus the corrections from the three loop graphs are vanishingly small and this effect persists to higher orders [9]. This is the reason why the two loop calculation of the viscosity exponent gives an answer surprisingly close to the experimental value.

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